

NATIONAL DISTRIBUTION & UNCERTAINTY

The national distribution of arsenic concentration measurements is the mixture of all the distributions from the individual sites:

$$F_{\text{National}} = \frac{1}{N} \sum_{\text{All sites } i} F_i,$$

where N is the total number of sites in the nation.

Similarly for our estimates:

$$\hat{F}_{\text{National}} = \sum_{\text{All sampled sites } i} w_i \hat{F}_i,$$

where w_i is a weight indicating how much of the nation is represented by site i .

However, \hat{F}_i is uncertain due to uncertainty in model parameters. The posterior uncertainty in \hat{F}_i is characterized by the many (equally likely) $\hat{F}_{i,j}$ obtained by evaluating \hat{F}_i with the parameters in MCMC sample j .

We can then compute the mean, cdf, median, 5th percentile, 95th percentile, etc. of the distribution of \hat{F}_i

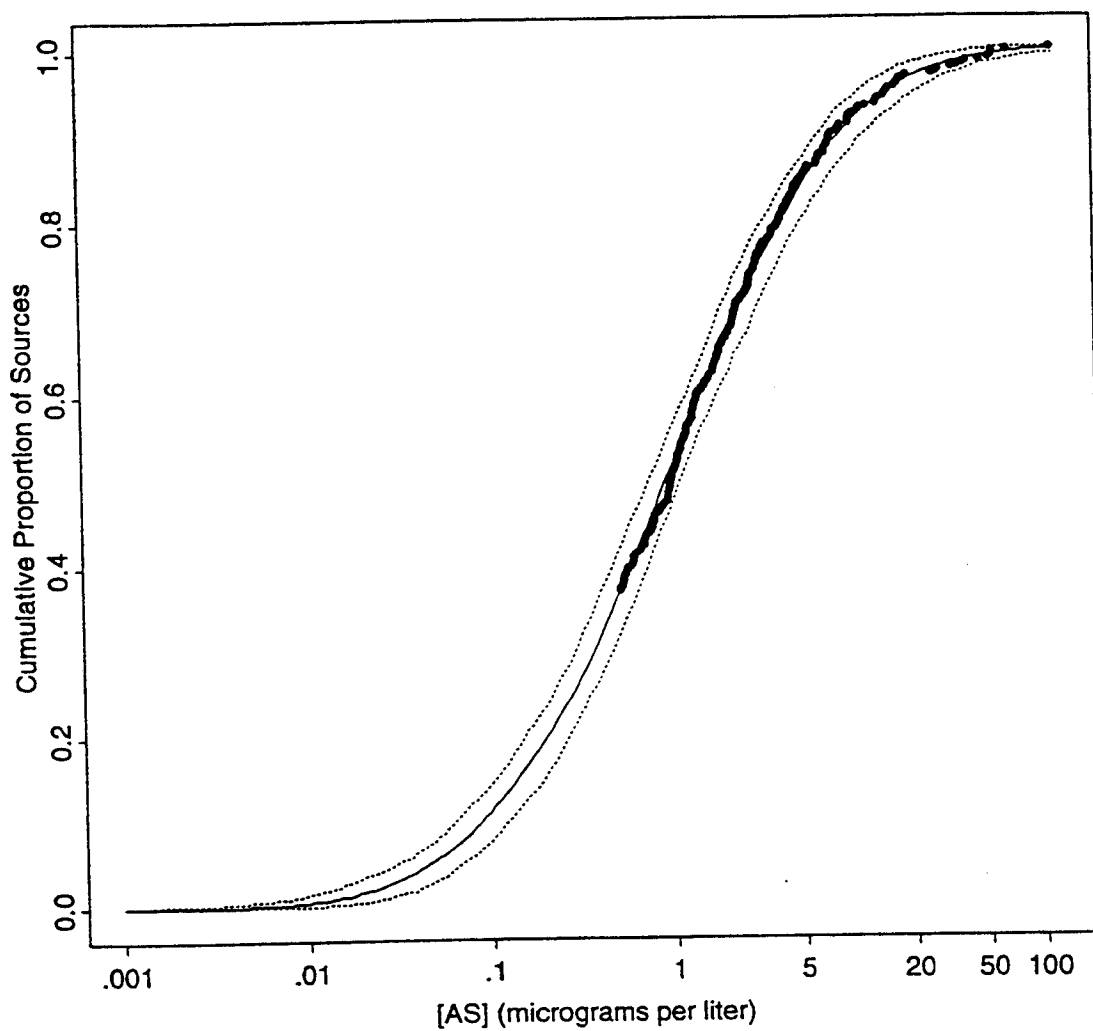


Figure 3: Posterior cumulative distribution function of national arsenic occurrence in source water with 90% credible bounds and uncensored NAOS data overlaid.

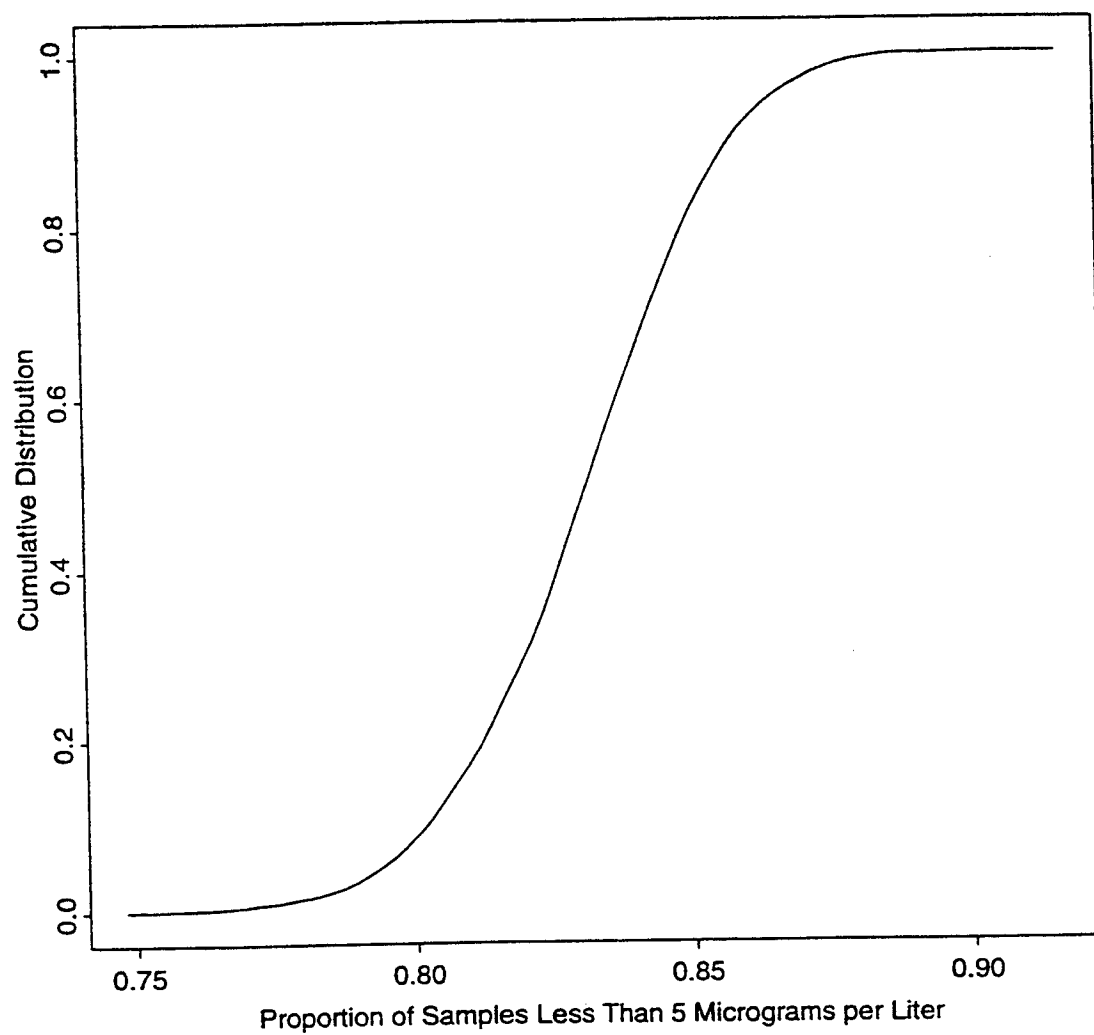


Figure 4: Posterior cumulative distribution function of the proportion of national arsenic occurrence less than 5 $\mu\text{g/L}$.

POSTERIOR ESTIMATES: Alternative Model

Supposing the γ should be positive, we kept all priors the same as before except took the prior for $\log(\gamma)$ to be $N(0, 10^2)$.

Parameter	P.M.	P.S.D.
μ_1	-2.78	0.55
μ_2	-3.17	0.51
μ_3	-3.27	0.49
μ_4	-1.42	0.44
μ_5	-1.50	0.48
μ_6	-0.71	0.54
μ_7	-1.04	0.48
σ^2	2.22	0.20
ψ	-1.94	0.65
τ^2	1.74	1.75
β	0.18	0.04
γ	0.03	0.07

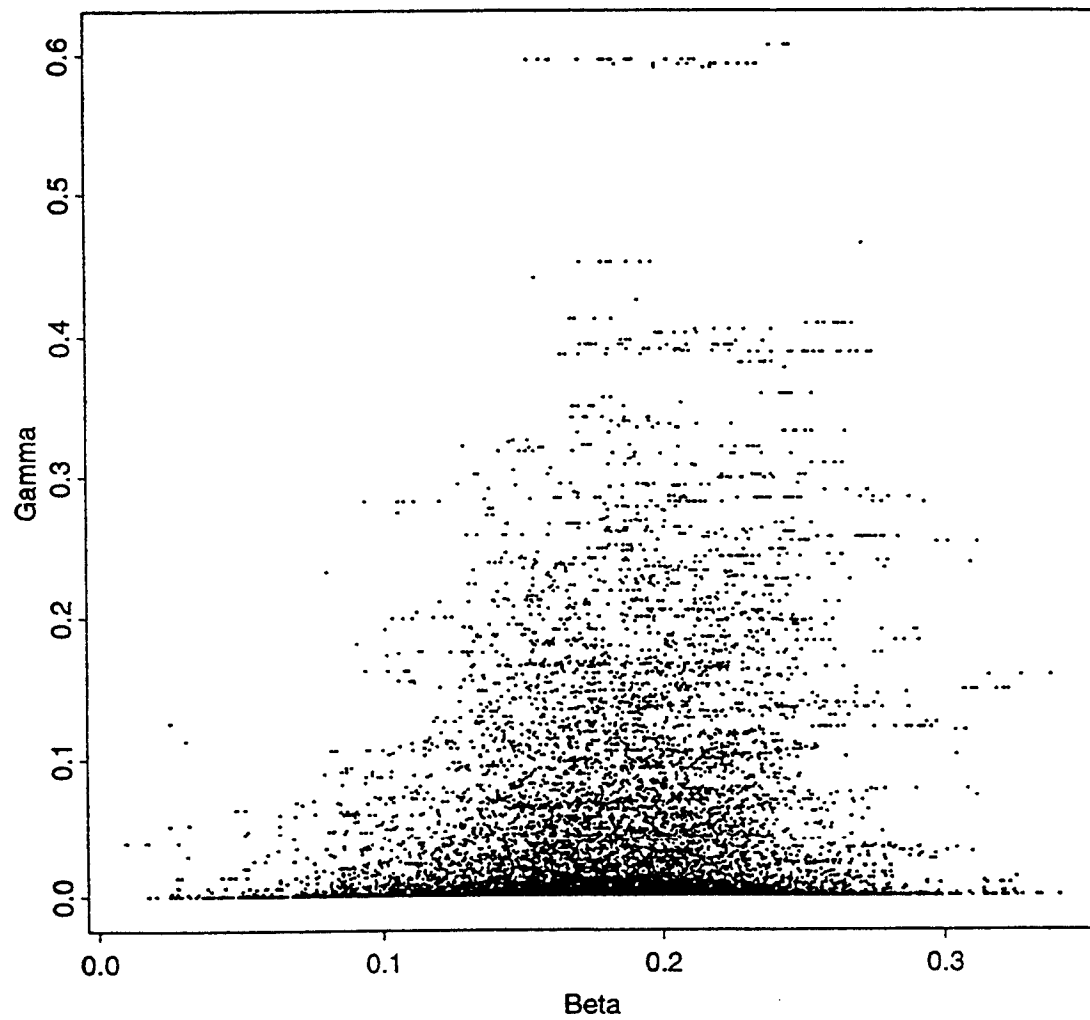


Figure 7: Scatterplot of γ versus β from a sample of size 30000 from the joint posterior distribution when γ is forced to be positive.

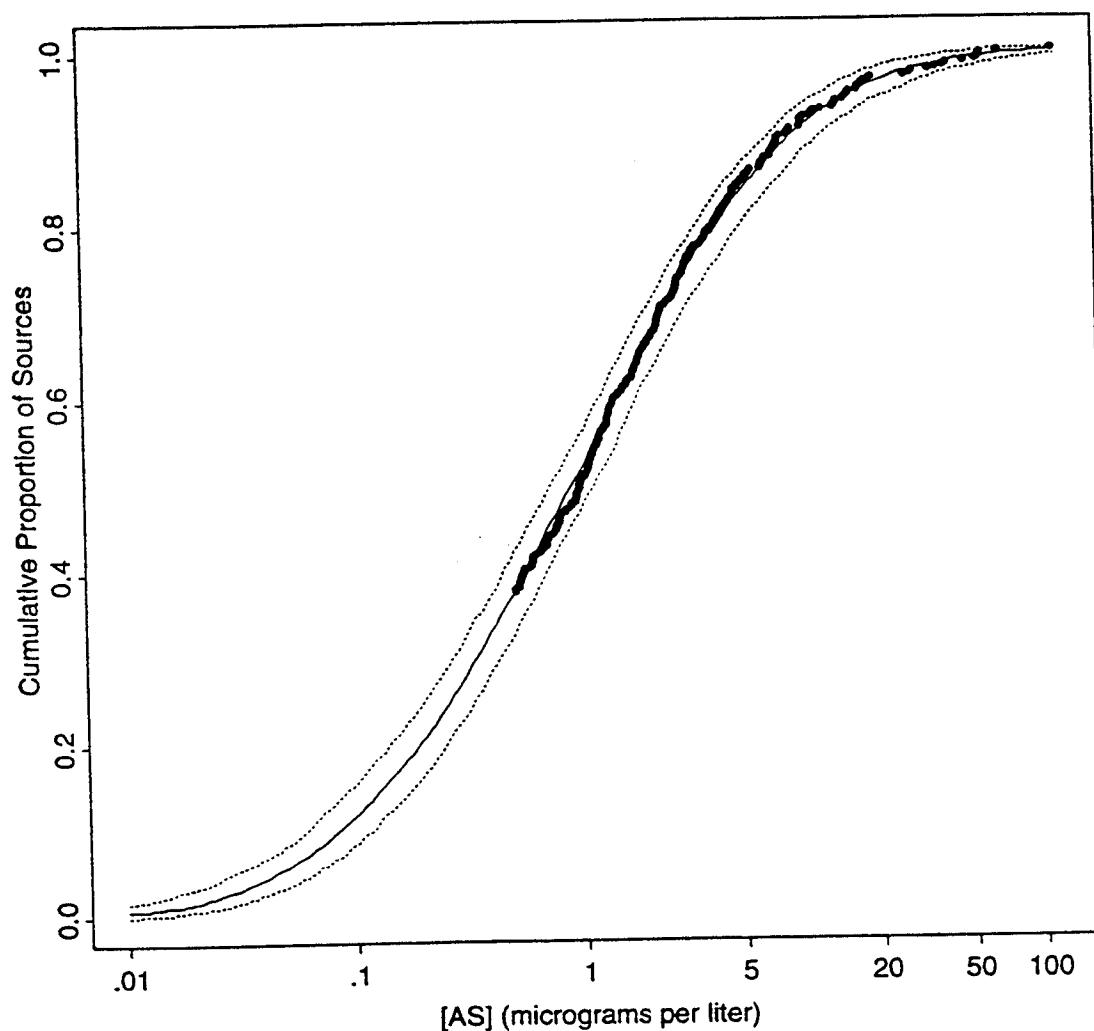


Figure 8: Posterior cumulative distribution function of national arsenic occurrence in source water with 90% credible bounds and uncensored NAOS data overlayed. Plot based on posterior when γ is forced to be positive.

SUMMARY

- Bayesian methodology provides a powerful method for characterizing variability and uncertainty in exposure factors
 - effect of alternative priors can be investigated in a diagnostic manner
 - though don't try this at home alone
(without a competent statistician)
- Probability distribution model with covariates provides insights, and a basis for extrapolation to other targeted populations or subpopulations.

Bayesian Analysis of Variability and Uncertainty of
Arsenic Concentrations in U.S. Public Water Supplies

John R. Lockwood
Mark J. Schervish
Department of Statistics

Patrick L. Gurian
Department of Engineering & Public Policy

Mitchell J. Small
Departments of Engineering & Public Policy
and Civil & Environmental Engineering

Carnegie Mellon University

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